ON THE CONSTRUCTION OF A MULTI-STAGE, MULTI-PERSON BUSINESS GAME

RICHARD BELLMAN*
The Rand Corporation, Santa Monica, California
CHARLES E CLARK and DONALD G MALCOLM
Booz, Allen & Hamilton, Chicago, Illinois
CLIFFORD J CRAFT and FRANC M. RICCIARDI
American Management Association, New York, New York

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The subject of this paper is a multi-stage, multi-person business game which will be used for executive training purposes by the American Management Association A discussion of the basic philosophy of game play, and of the many analytical, computational, and conceptual difficulties encountered in the construction of business games, is followed by a description of the game in question, as actually constructed and played, with particular attention to four features which, it is felt, merit consideration (1) Absence of an explicit criterion function (2) Principle of marginal change (3) Hidden formulas (4) Minimal computation The game (which, in a number of preliminary plays with top management participating, has met with a favorable reception) is outlined in some detail with a view to showing how it circumvents or overcomes a number of the obstacles described

IN THIS PAPER, we propose to discuss a number of questions connected with the interesting and significant problem of constructing business games that portray various aspects of economic and industrial interaction We shall combine a general discussion with a detailed study of a multi-stage, multi-person game with which we have had some experience over the past year

The Problem Stated

As is the case with other scientific tools—for example, digital computers—business games neither solve significant problems by themselves nor in any way replace the need for intelligent interpretation † However, it is the consensus that they are extremely valuable tools in the hands of experienced practitioners

Since many more groups are entering this field of activity—which shows definite signs of assuming a role in business planning and executive training—we feel that it will be a contribution to the development of this young

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[*] Consultant, Booz, Allen & Hamilton

[†] These remarks, of course, carry over in obvious fashion to the general study of simulation processes, of which the processes to be described constitute only a small part

art, first, to point out some of the pitfalls that may trap the unwary, second, to indicate some of the methods that can be used, and have been used, to circumvent these snares, and, finally, to describe some of the rewards at the end of the trail

A certain amount of effort has been devoted in recent years to the subject of economic games, and an even greater effort has been allocated to the closely allied domain of military games Yet little attention has been given to careful analyses of the basic philosophy motivating the construction of these models, with due regard to the numerous difficulties of a conceptual, analytical, and computational nature that arise in expected, and sometimes unexpected, places Moreover, since the very idea of a game simulating actual business practice is so attractive, and appeals so tremendously to the imagination, there is a real danger that the eager amateur may become so disillusioned upon experiencing unforeseen setbacks as to abandon the entire project

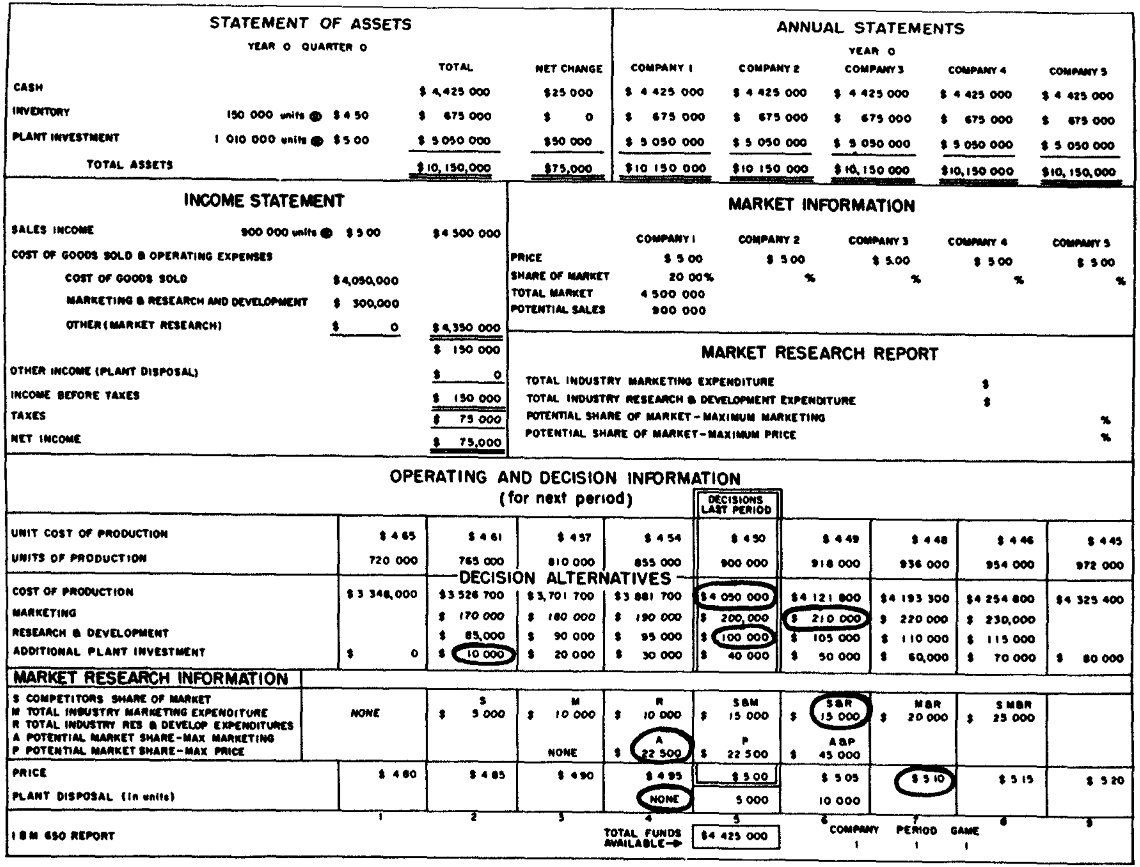
Although, in the sense that misery loves company, it is comforting to be assured that certain difficulties are well recognized, it is even better to know how to overcome these obstacles And, because we have had some degree of success in this direction, we feel that we have some positive suggestions to make toward the solution of a number of basic problems involved in the formulation of business games

A Frame of Reference

Perhaps a brief initial description of the particular game that has proved the inspiration for this paper will serve the useful purpose of creating a frame of reference for the ensuing discussion

The end product of the research to date is a game that simulates a competitive industrial situation wherein five companies are competing in a growing economy The usual objectives—desired share of market and growth in total assets—provide the motivation, as well as a rough measure of success, for the members of the five competing firms

Figure 1 shows the present game report form that is given to each company at the beginning of each 'play', it represents one-quarter of a year of operation As indicated, all companies begin the game with identical positions in regard to working funds, inventory, plant capacity, and price A single play involves decisions by each company team concerning the allocation of cash funds to the production, marketing, research and development, and plant investment programs of the company for the ensuing quarter The price of the product also must be specified In addition, it is possible to buy market information Each company makes its decisions by simply circling one of the possible choices for each category in the section "Operating and Decision Information" The sum of the



STATEMENT OF ASSETS

YEAR 0 QUARTER 0

		TOTAL	NET CHANGE
CASH		$ 4,425 000	$25 000
INVENTORY	150 000 units @ $ 4 50	$ 675 000	$ 0
PLANT INVESTMENT	1 010 000 units @ $ 5 00	$ 5 050 000	$50 000
TOTAL ASSETS		$10, 150,000	$75,000

ANNUAL STATEMENTS

YEAR 0

COMPANY 1	COMPANY 2	COMPANY 3	COMPANY 4	COMPANY 5
$ 4 425 000	$ 4 425 000	$ 4 425 000	$ 4 425 000	$ 4 425 000
$ 675 000	$ 675 000	$ 675 000	$ 675 000	$ 675 000
$ 5 050 000	$ 5 050 000	$ 5 050 000	$ 5 050 000	$ 5 050 000
$10 150 000	$10 150 000	$10, 150 000	$10,150 000	$10, 150,000

INCOME STATEMENT

SALES INCOME	900 000 units @ $ 5 00		$4 500 000
COST OF GOODS SOLD & OPERATING EXPENSES			
COST OF GOODS SOLD		$4,050,000	
MARKETING & RESEARCH AND DEVELOPMENT		$ 300,000	
OTHER (MARKET RESEARCH)		$ 0	$4,350 000
			$ 150 000
OTHER INCOME (PLANT DISPOSAL)			$ 0
INCOME BEFORE TAXES			$ 150 000
TAXES			$ 75 000
NET INCOME			$ 75,000

MARKET INFORMATION

	COMPANY 1	COMPANY 2	COMPANY 3	COMPANY 4	COMPANY 5
PRICE	$ 5 00	$ 5 00	$ 5.00	$ 5 00	$ 5 00
SHARE OF MARKET	20 00%	%	%	%	%
TOTAL MARKET	4 500 000				
POTENTIAL SALES	900 000				

MARKET RESEARCH REPORT

TOTAL INDUSTRY MARKETING EXPENDITURE	$	
TOTAL INDUSTRY RESEARCH & DEVELOPMENT EXPENDITURE	$	
POTENTIAL SHARE OF MARKET - MAXIMUM MARKETING		%
POTENTIAL SHARE OF MARKET - MAXIMUM PRICE		%

OPERATING AND DECISION INFORMATION
(for next period)

					DECISIONS LAST PERIOD				
UNIT COST OF PRODUCTION	$ 4 65	$ 4 61	$ 4 57	$ 4 54	$ 4 50	$ 4 49	$ 4 48	$ 4 46	$ 4 45
UNITS OF PRODUCTION	720 000	765 000	810 000	855 000	900 000	918 000	936 000	954 000	972 000
		DECISION ALTERNATIVES							
COST OF PRODUCTION	$ 3 348,000	$3 526 700	$3,701 700	$3 881 700	$4 050 000	$4 121 800	$4 193 300	$4 254 800	$4 325 400
MARKETING		$ 170 000	$ 180 000	$ 190 000	$ 200,000	$ 210 000	$ 220 000	$ 230,000	
RESEARCH & DEVELOPMENT		$ 85,000	$ 90 000	$ 95 000	$ 100 000	$ 105 000	$ 110 000	$ 115 000	
ADDITIONAL PLANT INVESTMENT	$ 0	$ 10 000	$ 20 000	$ 30 000	$ 40 000	$ 50 000	$ 60,000	$ 70 000	$ 80 000
MARKET RESEARCH INFORMATION									
S COMPETITORS SHARE OF MARKET M TOTAL INDUSTRY MARKETING EXPENDITURE R TOTAL INDUSTRY RES & DEVELOP EXPENDITURES A POTENTIAL MARKET SHARE-MAX MARKETING P POTENTIAL MARKET SHARE-MAX PRICE	NONE	S $ 5 000	M $ 10 000 NONE	R $ 10 000 A $ 22 500	S&M $ 15 000 P $ 22 500	S&R $ 15 000 A&P $ 45 000	M&R $ 20 000	S M&R $ 25 000	
PRICE	$ 4 80	$ 4 85	$ 4 90	$ 4 95	$ 5 00	$ 5 05	$ 5 10	$ 5 15	$ 5 20
PLANT DISPOSAL (in units)				NONE	5 000	10 000			
	1	2	3	4	5	6	7	8	9
I B M 650 REPORT				TOTAL FUNDS AVAILABLE→	$4 425 000	COMPANY 1	PERIOD 1	GAME 1	

Fig. 1. Present game report form

monies committed by these decisions must be equal to or less than the "Total Funds Available," shown in the box at the bottom of the form

The completed forms for all five companies (prepared in the usual traditions of industrial security) are handed to a control group that prepares the necessary information for submission to an IBM 650 computer * The computer has been programmed to calculate the effect of the decisions made by all the companies upon the share of the market each obtains and, also, each one's unit cost of production for the next quarter—a process that takes less than two minutes An IBM 407 tabulator is then used to print out a new form, with new decision levels for the next quarter or play of the game

This process is repeated to simulate as many years of operation as desired Forty periods, or ten years, is the most commonly used game length to date

Information concerning competitors' performance is available in much the same way as it is in the real world For example, price information is provided free each period, and 'annual statements' are provided every fourth quarter Other information—such as competitors' share of the market and the total expenditure by the industry for marketing, is purchased as desired Only then is this information printed out on a company's form

Order of Discussion

With these general remarks concerning the nature of the game with which we are concerned, let us summarize quickly the contents of the paper that follows This has been divided into a number of parts which may, to some extent, be read independently

In Part I, we consider some of the various reasons for constructing not only business games but multi-person, multi-stage games in general, beginning with the immediate objective of solving problems that arise in the business world We point out some of the various ways in which games can contribute in this direction, then we turn to a discussion of the many fascinating mathematical problems inevitably forced upon us in the course of our study Finally, we sketch the ways in which these games can be useful to the economist, to the industrial engineer, and to the psychologist, as well as to the university and the management educator in the teaching of mathematics, engineering, economics, and business administration

Part II is devoted to a partial enumeration and critical analysis of some problems that confront us at the very outset Since a number of these questions are closely related, there is necessarily a certain amount of repetition of essential points In Parts III and IV, we leave the realm of gener-

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* The game was adapted for electronic computation with considerable help from the International Business Machines Corporation under the supervision of JOEL M KIBBEE

alities and discuss the particular game we have constructed recently and have played extensively, attempting at each step to indicate the principles guiding us in the selection and rejection of various features Part V describes the mathematical structure of one of the early games, which remains basically unchanged even though a number of modifications have been made Lastly, Part VI presents some results taken from typical plays of the game in various stages of evolution

In a number of places our discussion is rather brief in order not to increase the size of the paper unduly It will be clear to the reader that in many of these places we are encountering fundamental questions common to any mathematical treatment of the physical world Tempting as it is to consider these in some detail, we feel that the result might well be to obscure the particular features of business games—that is, the topic of primary interest to us here

I OVER-ALL OBJECTIVES IN USE OF BUSINESS GAMES

It is clear that business games have a reasonable chance of being applied immediately to company planning and executive training It may not be quite so clear, however, that there are rewards, equal in their own way, awaiting the mathematician, the economist, the industrial engineer, and the psychologist Furthermore, there seem to be as yet untapped pedagogical possibilities in the use of games as classroom tools for courses in economics, business administration, and operations research

Application to the Business World

1 *Simulation* The basic hypothesis governing scientific research is that we can construct mathematical models of physical phenomena which will yield results in significant agreement with experimentally observed results Once we have a mathematical model in good agreement with observation in some directions, the further consequences of this model can be used to predict and to guide further experimentation With the advent of modern computing machines that permit us to consider processes of magnitudes undreamed of a decade or two ago, this same hypothesis has been gradually penetrating the business world

Despite the success of these mathematical methods in the physical world, there remain a host of problems, particularly in the engineering field, which defy the present level of mathematical ability To overcome them, we use a very simple idea In place of constructing mathematical models of a physical process, we construct actual models and proceed to determine the behavior of systems by direct experimentation Wind tunnels and towing tanks are two well-known examples of this simulation technique

Although there are a variety of classical techniques (such as the theory of differential equations and the calculus of variations) and a number of newly developed techniques (such as linear programming, dynamic programming, and sequential analysis) which can be applied to a cross section of questions, many of the most important business problems appear hopelessly beyond these devices at the moment

In the business world, a significant new factor—conveniently absent from most of the engineering problems appearing in these models—is the decision process, which involves the use of human beings and machines, rather than machines alone Many of the problems encountered are so involved that no simple simulation suffices. We must first construct a mathematical model, then construct a simulation process based upon it And many more problems arise to plague us in the construction of these business models than ever confronted an engineer The result is that the mathematician plays an essential role in designing the games and interpreting the results

Once we have constructed a satisfactory model, we are in the same position as an aeronautical engineer in possession of a wind tunnel Quite simply and rapidly we can observe the effects of parameter changes, test the effects of various policies, submit the system to random shocks, and generally perform the type of experimentation that in real life is usually too costly or impossible

2. *Abstraction* Since almost all significant problems in the economic sphere are too complex either for complete mathematical analysis or for complete simulation, it follows that we must content ourselves with making various types of approximations, or abstractions, in our analytical or our simulation techniques Our hope—and again it should be stressed that this is an article of faith—is to learn enough from the study of various combinations of analytical and simulation methods to be able to handle the actual problems

Making models, mathematical or otherwise, of complex systems is an art with a small amount of science to guide us Immediate success in this field is not to be expected, and a certain amount of failure is almost predestined

3 *Training in Decision-Making* If we are concerned with training executives to make decisions, or determining which executives are superior to others in this skill, there are several methods we can employ, separately or jointly

In the first place, we can train and test candidates in the specific areas in which they are going to operate In the second place, we can train them in general techniques of decision-making by means of processes which have no direct connection with their future field of expertness

On the whole, there seem to be a number of advantages to this latter approach in turning out versatile executives who have an over-all perspective of the art of decision-making It would seem that, once general principles have been grasped, it is relatively easy to adjust to any particular environment

Naturally, this principle can be carried to absurd lengths There is no substitute for a certain bedrock of knowledge and experience

Feedback to the Research Mathematician

1 *Variational Problems* The problem of determining the efficiency with which an economic system operates leads immediately to a variety of variational problems arising from a desire to maximize profit, minimize cost, or both Some of these can be resolved by means of classical techniques of calculus and the calculus of variations The majority, however, possess novel features requiring extensions of classical theory and the development of new techniques (*see* refs 1 and 2 for further discussion) Many of these problems, particularly those of a combinatorial type involved in scheduling theory, are complete beyond present-day mathematics and stand as challenges to our ingenuity (*see* refs 3 and 5)

In any case, it is as true here as in other aspects of the physical world that significant physical problems give rise to significant mathematical problems Conversely, research on significant mathematical problems will pay off in the solution of significant physical problems, which, in turn, will trigger further mathematical research, and so on

2 *Multi-Person Game Theory* The study of one-person processes, such as those involved in the allocation of resources or the scheduling of activities, leads to classes of variational problems similar in general structure to classical variational questions arising in mathematical physics The problems of maximizing profit or minimizing cost in many cases lead to mathematical questions identical in form with those arising from minimizing energy or maximizing volume In more modern applications of mathematical techniques, we deal with more complicated functions, but the basic principle of these one-person processes is the same maximize or minimize

When, however, we begin the study of multi-person processes involving individuals engaged in competitive endeavor, the questions that arise are conceptually different from those of classical theory The von Neumann-Morgenstern theory of games, superseding an earlier attempt of E Borel, represents a tremendous advance in laying down the foundations of two-person games The theory, however, is only satisfying in the treatment of two-person zero-sum games The most important applications involve non-zero-sum games, games in which the competitors have different utility functions and, in many cases, there are three or more competitors

Most likely, despite a number of highly ingenious efforts (*cf* von Neumann-Morgenstern,[10] Nash,[7] and Shapley[9]), there will never be a unified theory, but rather a set of incompatible theories adjusted to a variety of particular situations (*cf*, in this regard, McKinsey[6])

It is to be hoped, then, that a thorough study of particular multiperson games, beginning with an analysis of their formulation and culminating in a study of actual play, will furnish clues leading to more satisfactory theories of N-person games

3 *Learning and Prediction Theory* The study of multi-stage decision processes leads in a natural way to the study of realistic processes where we must simultaneously make decisions and explore the partially unknown structure of the underlying system The problem area is so huge and unexplored that the study of particular processes serves a vital focusing role This is, of course, a basic virtue of all mathematical models of significant processes occurring in the real world

In regard to prediction theory, despite the enormous advances contained in the work of Wald,[11] and Wiener-Kolmogoroff,[12] quite basic problems defy successful analysis at the moment In the field of learning theory, a theory of paramount interest to the psychologist, the statistician, and through them to the mathematician, even the simplest-appearing problems baffle us and seem to escape precise formulation *

Here, again, particular processes will light the way to a coherent theory

4 *Computational Techniques* Once we have constructed a mathematical model of a process, to be treated by some combination of analytic and simulation techniques, we are well on our way in our investigation of a system However, the final objective of interpretation and understanding cannot be achieved until we have methods for obtaining output numbers from input numbers Furthermore, in order to experiment successfully, we must be able to perform calculations quickly and accurately

Problems of this type are highlighted in the study of multi-stage processes and, particularly, in the study of multi-stage, multi-person processes with their combinatorial overtones The experience gained in the successful treatment of one process can usually be carried over to the treatment of others Again, particular processes serve a useful triggering role

In most cases, the use of a digital computer is assumed

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* A discussion of some aspects of learning processes will be found in Robbins,[8] *cf*, also Bush-Mosteller [4]

5 *Policies* One way to avoid the multi-dimensional wilderness of multi-stage processes is to focus upon the concept of a 'policy' or 'strategy ' Games and other types of simulation processes serve as excellent proving grounds for the study of the effects of various policies Reciprocally, the study of these games yields information concerning efficient policies

One of the main purposes of the game we have developed is to force the players to think in terms of policies and long-range effects

6 *Interaction Theory* There is—to repeat—a science of model-building, which is to say the construction of mathematical idealizations of physical processes Hence the ability gained in one domain in translating such elusive concepts as *information*, *competition*, *efficiency*, and *learning* into clean-cut mathematical formulas permitting quantitative evaluation can readily be carried over into other domains We thus gain a foothold on the terrain of general interaction theory

The study of multi-stage, multi-person decision processes draws heavily upon these skills, and the outside world is capable of furnishing an unlimited quantity of interesting and important processes, each with its special features to intrigue the mathematician

Feedback to the Economist

1 *Interaction Study* The construction of a mathematical model in any field serves the essential role of crystallizing thinking Vague, general statements must be reduced to precise quantitative statements the consequences of which can be tested and evaluated In particular, we are led to studies of cause and effect, and of payoff and motivation

2 *Data Collection* The construction of a mathematical model tells us what information is required for further study, and in what form data should be collected This is one of the major purposes of a study of this type, and sometimes its sole purpose, but it may also help us dispose of unnecessary accumulations of data—a problem that is becoming of greater and greater significance

3 *Key Variables* In the course of experimentation, we can hope to observe the relative importance of various factors If we note that some variables play an unimportant role, we can eliminate them from the model and thus simplify the mathematical and economic analysis considerably On the other hand, unsatisfactory behavior may force us to the conclusion that we may have lumped some variables uncritically or neglected others

Questions of this type are very difficult to resolve analytically

Feedback to the Psychologist

1 *Decision-Making* A problem of prime concern to the psychologist is the study of the why's and wherefore's of various types of decision-making Although the standard way to learn about these processes is to carry out experiments, a common difficulty is that the motivation in these experiments is never powerful enough to ensure that the subjects act in a manner reflecting their behavior under actual situations

In playing our game, we have been in a unique position insofar as the selection of players is concerned Choosing top management, strongly actuated by prestige considerations and intellectual curiosity, we have had an excellent opportunity for significant psychological observation of the decision-making processes of a very important stratum of American life By changing the information pattern, the duration of play, and so forth, we can carry out psychological experimentation, as well as gain in mathematical and economic interpretation

2 *Group Interaction* In a multi-person game of this type, the information pattern is such that the kind of game that is actually played depends to a considerable extent upon the particular players selected We thus have a means of observing group interaction, controllable to some degree by varying the information given each team concerning the other teams' positions

3 *Learning Processes* In the study of the players' behavior as they attempt to understand the structure of the game, and to predict the moves of their competitors, we have a ready-made psychological laboratory Furthermore, we are dealing with people of stature and responsibility engaged in the solution of problems similar to those that confront them in their actual lives, rather than volunteers engaged in make-believe situations playing for pennies

It need not be emphasized that there will be the usual difficulties of reproducibility and statistical analysis attendant upon all psychological studies It is to be hoped that the records of thousands of games of this type will disclose certain structural properties This would seem to offer the blueprint for an ambitious program of psychological research

Feedback to the Industrial Engineer

The industrial engineer must embody within himself something of the mathematician, something of the economist, and something of the psychologist Among his many other duties, it will be the industrial engineer's job to criticize, implement, or install the new procedures, controls, and so forth, that may be suggested as operational changes by the results of psychological research It then follows that the study of business games furnishes an excellent means of tying these threads together and providing a certain amount of synthetic experience

Feedback to the Classroom

The fact that games of this type furnish excellent motivation and material for courses in mathematics, operations analysis, economics, and business administration needs no amplification

II PROBLEMS—'SNAGS IN THE YARN'

A NUMBER of unpleasant problems arise as soon as we descend from the qualitative to the quantitative and approach the problem of constructing actual business games

What Constitutes Optimal Play?

Two immediate difficulties confront us in attempting to define optimal play The first is common to all types of decision processes and arises in connection with the construction of any mathematical model, while the second is characteristic of two-person and, to an even greater degree, N-person decision processes

At first glance, it would appear to be an easy task to assess the degree of success or failure of a business enterprise, relying upon such well-known indicators as total assets, profit, and share of market A little thought, however, will reveal the fact that these are static data, and not entirely trustworthy in evaluating a dynamic process Therefore, in addition to this information, which describes the state of the process, we like to know something about the rate of change of total assets, the rate of change of profit, and so on In other words, the history of the process may be important, and very often is Granted that we possess all the desired information, there is still the problem of evaluation

Many of the factors mentioned are incommensurable, or quasi-incommensurable By this we mean that there is no convenient yardstick for converting a statistic for the one into a statistic for the other This, combined with a distinct lack of unanimity among business executives, economists, industrial engineers, mathematicians, and the like as to how to weight these factors, renders the construction of a criterion function that serves the purpose of evaluating policies a matter of great difficulty In fact, one of the functions of a business game of this type is to furnish these criteria

A further difficulty that arises is due to what may be called an 'end effect ' If we start an economic process at this time, it may seem reasonable to consider maximization of total profit over the next ten years as a sensible goal However, after nine years have passed, we certainly do not wish to continue as if the only goal were to maximize profit over the coming year

There are several commonly used ways of circumventing this end effect, but they smack more of mathematical convenience than any intuitive operational concept The basic idea is to discount the future in some systematic fashion This is closely connected with prediction theory and shares the usual difficulties

Let us now consider a characteristic difficulty of multi-person games In the one-person process, arising from a simulation or programming problem, once a criterion function has been decided on, we possess a simple means of determining optimal play—it is play that maximizes the criterion function However, in the multi-person process, no such simple-minded optimization is in general possible

To illustrate this, let us consider the two-person process first Further, let us take the simplest case, where the players are in direct competition, so that one's loss is the other's gain Let us assume, as is natural, that there is an interaction between the players, which means that the return to each is dependent upon the actions of both It follows that neither side can maximize without paying attention to the decisions of the competitor

The von Neumann-Morgenstern theory of games suggests how to resolve this apparent circularity It is shown that a certain value, a number, can be attached to the game, having the property that one player can guarantee at least that return if he plays properly* and the other player can guarantee not losing more than that quantity

Let us now add two realistic features

1. The players are not in direct competition
2. The players possess different utility scales, i e , they have different estimates of what constitutes optimal play

There exist several proposed theories to treat these more general processes, none, however, is of any universal acceptance Thus we have no unique way of determining optimal play for these realistic processes, even if we can decide on an appropriate criterion function

N-person games of any type, $N \geqq 3$, are virtually impossible to cope with analytically Again there are several proposed theories (von Neumann-Morgenstern,[10] Nash,[7] Shapley,[9]), but none is uniformly satisfactory This fact is significant, and must be emphasized, since it is not at all intuitive and contradicts a number of beliefs many of us hold dear

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* The concept of 'proper play' is not a simple one, involving as it does the ideas of randomization and average return The classic work on the subject is von Neumann-Morgenstern,[10] while the most entertaining and readable account of the fundamental ideas is contained in Williams [13]

In a one-person process, with a definite criterion for measuring the effect of a sequence of decisions, there is a unique set of policies that we can call optimal and that maximize the criterion function In various types of two-person games, as we have mentioned, each player possesses a certain set of strategies that guarantee a certain average return, regardless of what the other player does Furthermore, deviation from these strategies may be expensive

In multi-person games, as well as in the general two-person games, such sets of optimal strategies need not, and in the majority of cases, do not, exist The consequence is that a set of policies that works well in a game involving one set of players may be disastrous in a game played under the same rules but involving a different set of players with different philosophies as to optimal play *

The fact that we cannot determine optimal play does not destroy the usefulness of these games It actually makes them more valuable, since they can be used to explore the effects of various classes of policies Generally speaking, this is much more important information than a knowledge of optimal play, which in many cases may be far too complicated in structure ever to use

This last idea is an important point which must continually be kept in mind Although it is not desirable for an optimal policy to possess a higher degree of complication than the structure of the model itself, it is difficult to determine when this situation does indeed exist

What Are the Effects of Decisions?

In the course of play each player is required to make decisions concerning the allocation of money for advertising purposes, for research and development, and for production, determining both output and increase or decrease in plant capacity, and finally to fix the price of the item

The choice of advertising budget, research and development budget, and price to some extent determine, as we know, the share of the market But how? Certain qualitative features can be seen easily—for example, the fact that increasing the first two allocations and decreasing the price increases the share of the market, and conversely However, quantitative knowledge in this area simply does not exist There are no experts to refer to, no reports to read

At first sight, this depressing state of affairs would seem to militate against the whole idea of a business game But there is no reason to be too discouraged After all, decisions are continually being made in actual

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* This is a well-known fact as far as card games are concerned Optimal play must be a combination of certain basic principles and information concerning the psychology of the opponents Interestingly enough, this is even true of a game like chess

business operation without precise knowledge of these effects Actually, we have turned this very real difficulty into an advantage

How Detailed and How Realistic Should the Model Be?

We start out with the knowledge that the only accurate model of a process is the process itself, and hence that any mathematical or simulation model is an approximation of a process that is itself an approximation We see then that we are always faced with the difficult question of determining the degree of realism, and thus complication, that we wish our model to possess

The type of model to be constructed should obviously depend upon the depth of the answers we wish to obtain Frequently, however, a certain amount of trial and error is required in order to determine the proper level of complexity Here it is necessary to tread a very narrow path between the danger of oversimplification and the morass of overcomplication If the model is oversimplified, it lacks sensitivity and we cannot distinguish between large classes of policies, if the model is overcomplicated, we will not be able to isolate cause and effect

Let us further note that complication in the model usually increases computational labor in a nonlinear way In other words, what appears to be one additional factor may increase computing time by an appreciable factor, such as a fourth or a third

Finally, it should be emphasized that an increase in complexity does not necessarily entail an increase in accuracy, and that actually the reverse may occur. If, for example, we set up a model that involves ten different activities requiring, say, the solution of systems of ten linear equations in ten unknowns, we obtain certain estimates of parameters that determine optimal policies Carried away by the success of this operation, we may attempt to set up a model that involves a hundred different activities involving the solution of a system of a hundred linear equations in a hundred unknowns. The estimates for the parameters obtained from this more sophisticated model may be completely nonsensical, owing to the fact that the large number of numerical operations involved in the solution of large systems, each involving round-off error, may completely overwhelm the input data, accurate to only a few significant figures Furthermore, the model can very easily become *less stable*, as its size is increased, rather than *more stable* All these factors must be carefully considered before large-scale models are constructed and the computing machines are set in operation

A particular case of the problem of isolating cause and effect is the question of deciding between *deterministic* versus *stochastic* models By a deterministic model, we mean one in which the outcome of every decision is uniquely determined by the decision, although the mechanism may perhaps not be known to the player By a stochastic model, we mean one in which the outcome of a decision is a random variable, with a distribution function that may be known or unknown to the player The temptation in many cases is to construct stochastic-model processes in the hope that they reflect the actual business operation more accurately

The difficulty, however, arises in evaluating the outcomes Is a good result the product of a superior policy, or is it due merely to a fortuitous chain of events? Is a poor result the product of an inferior policy, or is it due merely to a run of hard luck? Unless a stochastic process is run a sufficient number of times, we may not be able to answer these questions readily Consequently, if we are interested in testing policies, it may be more efficient to use deterministic processes initially

Let us note in passing that, if the underlying process is genuinely stochastic, there is the usual difficulty of choosing a criterion function for a process that may be carried out only once, or at best a few times This is one of the fundamental difficulties of the theory of games Its resolution depends upon one's personal philosophy

As a general rule of thumb, it is far better to start with apparently simple models, and gradually to increase the complexity of the model on the basis of experience in actual play, than to mire oneself in an unwieldy, complicated model at the outset It is quite amazing to see the depth of optimal policies possessed by apparently simple processes *

How Difficult Should It Be to Make Decisions?

In constructing a game of this type, which is intended for a fairly large audience, a great deal of thought must be devoted to (1) learning the game, (2) the information pattern, and (3) decision-making These three problems are, of course, closely interrelated

1 *Learning the Game* There are two phases to learning the game The first consists of understanding the basic structure of the game and its objectives, while the second consists of knowledge of the individual moves It should be repeatedly stressed that simplicity must be the principal theme If not, it is very difficult—if not impossible—to pursue policies and observe their outcome Since simple decisions at each stage of a multi-stage process can combine to yield exceedingly complicated policies, there is no need for games of intricate local structure

2 *Information Pattern* The decisions of the players will depend, once the rules of the game have been detailed, upon the information available to them concerning the state of the process Thus the type of game

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* The Japanese game of *go* is a good example

that is played depends to a large extent upon the information pattern This is a valuable fact to keep in mind, since it furnishes the game-maker with a fairly simple way of modifying and altering the game

There are three aspects to the information pattern (a) The player's own position, (b) the position of the other players, and (c) the structure of the game, i e , what makes the wheels turn It is both realistic—and desirable—to have concealed information concerning any or all of these factors

3 *Decision-Making* Once we have covered the first two points—learning the game and the nature of the information pattern—we are faced with the problem of deciding upon the time to be allocated to the player in making decisions, and the tools to be employed for this purpose Since we wish to train the player to think in terms of fundamentals and to take long-term and over-all views, we must prevent him from being bogged down in arithmetical computation, keep him from being distracted by detail, and force him to think in terms of essentials This is not an easy objective to attain, since it is not at all simple to distinguish between unessential detail and significant information, particularly at the beginning of the game

In designing our game, we were careful to avoid putting a premium upon mathematic training or upon ability in rapid arithmetic Generally, the emphasis is upon correctness of principles, rather than quickness of action There is no reason, of course, why particular games should not be designed to test rapid-fire abilities, but it should be understood that in the majority of decision processes there is no shortage of time

Three Crucial Points

Three major points are crucial in deciding the success of a game

1 *Stability* In the course of a football season, a superior team can be upset by an inferior team, or lose a close game on a fluke play, a blocked kick, a fumble, or a desperation pass However, when we examine the records of games over a season, it is readily seen that the groups with better-trained players and superior teamwork have consistently better records

The same principle holds in the construction of these games The purpose is to emphasize the worth of sound principles and long-term planning Consequently, we want to be sure that the state of the game cannot vary widely from stage to stage as a consequence of fluke moves by the players This is what we mean by *stability*

We do not mean to imply that the real world does not contain examples of brilliant coups which have rescued seemingly hopeless situations

We do maintain, nonetheless, that they are the exception rather than the rule Before one attempts brilliance, one must know sound moves A foundation of training in basic principles is the proper springboard for innovation

2 *Elasticity* If we circumscribe decisions in such a way as to ensure stability, we may find that the game has become insensitive to even gross changes of policy The Scylla of extreme sensitivity or unstability is balanced by the Charybdis of extreme insensitivity or *inelasticity*, and we must pursue a careful path between these two Although it is not an easy task, a great deal can be done upon observing the play of the game Trial and error and patient, if profane, calculation play essential roles.

3 *Gimmicks* Since the purpose of the game is training, or the solution of actual problems, we must make sure that the players act as they would in the actual business situation, rather than seize upon special features of the game In particular, there must not be extreme policies, obviously unrealistic in the actual process, which are successful In other words, there must be no gimmicks

Unfortunately, it is not so easy to guard against gimmick policies as one might think Checks and balances against extreme policies, which exist in the real world, may very often be omitted in the necessarily small-scale model we build Quite often it is easier to build in an artificial guard against extreme policies than to include the actual mechanism that performs this vital role in the realistic process

Limitations of the Game—'Caveat Ludeator'

Finally, the game must not be taken too seriously No matter how accurately it reflects the principles discussed, and no matter how realistic its optimal policies may seem, we must constantly remember that it is, after all, only an approximation to reality

Let us point out, also, that the game does not touch many areas in which top management must make crucial decisions Some of these are

1 Innovation and technological advances
2. Environmental and, in particular, governmental influences
3. Catastrophes
4. Substitution of other products by the customer
5. Mergers and coalitions
6. Labor-management problems

III. GENERAL DESCRIPTION OF A MULTI-PERSON, MULTI-STAGE GAME

THE BASIC situation in our game is that of a number of firms producing a single item competing for a known consumer market Each firm, which is represented by a team of one or more players, possesses the following information concerning its own position at each state of the process

1. Total sales in units and dollars over the preceding time period
2. Price of the item during the preceding time period
3. Opening inventory level for the next period
4. Maximum productive rate for the next time period, and actual productive rate for the preceding time period
5. Unit cost of production
6. Share of market during the preceding time period
7. Allocation of the total budget to marketing, research and development, and additional plant investment during the last period
8. Total working funds available for allocation during the next period

In addition, it possesses a certain amount of information concerning its competitors—for example, their prices and perhaps their shares of the market

On the basis of this information and past history, which the players are allowed and encouraged to keep, a number of decisions must be made governing the play over the next period These involve the determination of

1. Price
2. Marketing budget
3. Research and development budget
4. Rate of production
5. Plant investment

A basic restriction on allocations is that no borrowing is allowed, which means that all budgetary allocations must be covered by working funds on hand

Concerning the effects of their decisions, the players are given only the following obvious qualitative information

1 Increase in productive capacity increases maximum production rate
2. Increase in productive capacity decreases unit cost
3. Increase in utilization of productive capacity (i e , ratio of actual productive rate to productive capacity) decreases unit cost
4. Increase in research and development decreases unit cost, and increase in research and development relative to competition increases attractiveness of product
5. Increase in marketing expenditure relative to competition increases attractiveness of product
6. Increase in price relative to competition decreases attractiveness of product
7. Attractiveness, which depends upon price, marketing, and research and development, determines share of market

The market consists of a known total demand for the item which increases at a certain known rate per stage

This process continues for a fixed number of stages, with each team making its decisions so as to *optimize*

IV. SPECIAL FEATURES OF THE GAME

A NUMBER of special features which we have built into our game circumvent or resolve, partially or wholly, some of the traditional difficulties which have been pointed out These special features, which we feel represent our contribution to the art, are as follows

A Absence of an explicit criterion function
B Principle of marginal change
C Hidden formulas
D Minimal computation
E Deterministic yet quasi-stochastic nature

A. Absence of an Explicit Criterion Function

Because of the confusion and contradiction that are present in the statement of any explicit criterion function—for example, maximum profit over a fixed number of stages—it was decided to eliminate, not only any analytical definition of the criterion function, but even any explicit mention of a particular goal Instead, the players are told to play the game as if it were an actual business operation The burden is then doubly shifted The game must be realistic enough to motivate this behavior, and the players must be mature enough, and sufficiently interested, to pursue this course

We might say that, in our experience with teams drawn from top management and academic ranks, we have had no difficulty in this direction The imprecisely defined situation is, after all, realistic In any actual competitive situation, the payoff is not well defined, and there exists no clean-cut evaluation of a policy

Observe that this type of criterion, implicit rather than explicit, avoids the end effect described earlier and to some extent diminishes the possibility of optimal policies of the gimmick variety

B. Principle of Marginal Change

In realistic processes occurring in the economic sphere, there is very often a time interval between a decision and the effect of that decision Thus the construction of new plant capacity may take a year, planning an advertising campaign may take six months, and the results of money devoted to research and development may take years to show up These *time lags* necessitate long-term planning If we attempt to take this retardation into account, we encounter a formidable difficulty so far as the information pattern is concerned, and a further difficulty in connection with the stability of the process

At the present time, we have as state variables a certain set of quantities If we allow a time lag, the information pattern must contain not only the present state of the system, but also a good deal of the past history, the amount being dependent upon the length of the lag Not only is this greater amount of detail more difficult for the player to keep track of and assimilate into the decision process, but it also greatly increases the computational effect required to determine the effects of decisions at each stage and hence the time required per stage This difficulty is by no means insurmountable, particularly with modern digital computers, although careful thought should be given to whether the degree of complication introduced by time lags is worth the gained realism in view of the many costs to both the players and the directors of the game However, there is also the discontinuity in the play of the game brought about by a sudden increase in productive capacity, a sudden spurt of advertising activity, and similar behavior

To counter both these difficulties, we have introduced the constraint of marginal change By this we mean that none of the state variables can be affected by any decision at any stage by more than a certain percentage of its value, which depends upon the individual state variable and upon the stage of the game This constraint

1. Automatically insures a certain degree of stability
2. Forces long-term planning, since major changes require a large number of stages
3. Simplifies decision-making on the part of the player
4. Combined with the idea of discrete change, greatly simplifies decision-making, computation, and tabulation of results

Since realistic business processes do contain discontinuous features of the type described, it is important to know whether the absence of time lag is a major defect We have, in fact, shown by mathematical analysis that in a process of sufficient length the effect of time lag washes out

C. Hidden Formulas

The fact that in business operations there are no precise relations connecting allocations and monetary return may be taken as caused either by ignorance or by the stochastic mechanism of the actual processes involved Consequently, we decided not to disclose the formulas used by the computer to determine the outcomes of decisions, but to give the teams only the structural information concerning the process outlined here in Part III

Apart from its realistic aspects, this decision

1. Forces the players to think in terms of general concepts and policies
2. Equalizes the mathematical level of the teams
3 Simplifies decision-making and reduces the time required per stage
4. Prevents gimmick policies to some extent

D. Minimal Computation

If a great deal of value is to be derived from playing the game, it is essential, as we have said, that the maximum effort of each team be devoted to thinking in terms of basic policies, and that a minimum of effort be devoted to inessential calculation In particular, the amount of arithmetic work should be kept at the very minimum

Especially helpful in this context is the concept of marginal change, which automatically forces each team to consider at each stage only a small set of decisions associated with its current position Thus, for example, if the price of the item chosen by a particular team is $5 00, and we allow a maximum change of 4 per cent in the price over any period, there is room for choosing a price between $4 80 and $5 20 But a choice of 41 different prices, which we obtain upon permitting changes of a penny, is still too free, particularly in view of the fact that the team possesses a slight rational basis for a choice between $5 11 and $5 12, say, as a consequence of the hidden formulas discussed Hence, we introduce as an additional constraint the condition that, within the 4 per cent range, the price can vary only by multiples of 5 cents The team thus has a choice of nine new prices

$4 80, 4 85, 4 90, 4 95, 5 00, 5 05, 5 10, 5 15, 5 20

Furthermore, this range of prices is tabulated for the team at each stage by the digital computer, together with a table of other admissible allocations and decisions

This further principle of discrete steps, combined with a tabulation by the computer of possibilities for each team at each stage, considerably reduces the arithmetic labor of each team and almost completely eliminates the possibility of trivial computational errors on the part of the players

E Deterministic Yet Quasi-Stochastic Nature

In order to spotlight the effects of policies, and prevent to some extent the onus of failure from being shifted from the player to chance, it was decided to use a deterministic model

Nevertheless, the game possesses stochastic features Since there are no guiding rules for optimal play, the type of game that develops depends to an overwhelming extent upon the composition of the teams—which is to say the philosophies and psychologies of the individual players The chance element is actually present in the random choice of players

The same player, pursuing the same policy, can do very well in one run of the game and very poorly in the next Thus the game possesses the desirable quality of forcing the player to be *flexible* and adjust his policies to meet actual competitive conditions There would be little difficulty in converting the game from a deterministic to a stochastic model

V MATHEMATICAL STRUCTURE OF THE GAME

ALTHOUGH the players do not need the analytical form of the equations describing the interaction of the decisions, the digital computer does The two basic quantities are the attractiveness of the product, which determines the share of the market, and unit cost of production

Attractiveness

Since we are very much concerned with the stability of the process, we begin by converting two of the cash allocations into ratios in order to decrease their sensitivity Let a_i be the marketing expenditure of the ith team at a particular stage We next compute the new quantities

$$a_i' = a_i / \sum_i a_i$$

Let r_i be the total allocation of the ith team for research and development over the last four stages (including the current state) Then

$$r_i' = r_i / \sum_i r_i$$

We now define the attractiveness, denoted by A_i, of the product produced by the ith team as a function of the three quantities a_i', r_i', and p_i, the price of the item set by the ith team,

$$A_i = f(a_i', r_i', p_i)$$

Once A_i is determined, the fraction of the market purchasing the item of the ith team is given by

$$f_i = A_i / \sum_i A_i$$

Hence, if N is the size of the market, the income from sales to the ith player is given by

$$R_i = p_i f_i N,$$

provided that $f_i N \leqq q_i$, the quantity of items produced over the last period plus those in inventory.

It remains to determine the function $f(a_i', r_i', p_i)$ As pointed out, we have little basis for any analytical description of the function Consequently, we decided to choose the simplest function possessing the correct qualitative features

To begin with, we want $A_i>0$ for all values of a_i', r_i', p_i Therefore, f takes the form

$$f(a_i', r_i', p_i)=\exp[g(a_i', r_i', p_i)]$$

The simplest form for g is a linear function

$$g(a_i', r_i', p_i)=c_1\, a_i'+c_2\, r_i'-c_3\, p_i,$$

where c_1, c_2, $c_3>0$ The function f now has the correct qualitative behavior

We found, however, that this function was still too sensitive to changes As a result, we added a buffer constant c_4 and let f have the form

$$f=c_4+\exp[c_1\, a_i'+c_2\, r_i'-c_3\, p_i]$$

Unit Cost of Production

In designing a formula expressing the dependence of the unit cost of production upon the decisions made at each stage, we took this quantity to depend upon

1. The maximum potential rate of production, i e , the plant capacity
2. The actual rate of production
3. The research and development budget

Qualitatively, it was felt that the unit cost of production should decrease as the actual rate of production approaches the maximum rate of production, and should decrease as the research and development budget increases and as plant capacity increases

After a certain amount of trial and error, we decided upon the following formulas for u, the unit cost of production

$$u=c_5+(c_6+c_7\, M)/(1+c_8\, m)+\max[c_9/(1+c_{10}r), c_{11}]+c_{12}/M,$$

where the c_i, $i=5, 6, \cdot\ \ , 13$, are positive constants, and M is the maximum potential rate of production, m the actual rate of production

Here the purpose of the term c_5 is to keep the unit cost from being too sensitive to changes, the second term measures the effect of partial utilization of the plant, the third term represents the effect of research and production in turning out a more easily made product, and the fourth term relates the effect of plant capacity

WE HAVE PRESENTED the simplest versions of these functions here to indicate the ideas guiding our choices Actually, in more recent plays of the game, we have modified these functions in a number of ways both to preserve stability in the beginning of the process and to alter the elasticity of the market in the middle and end of the game

Once the construction of this game is clear, it is relatively easy to design more complex games involving the production, sale, and distribution of a number of products

VI SOME TYPICAL PLAYS

IN ALL, the game has been played some twenty-five times, however, four 'bench-mark' plays will suffice to illustrate the way in which the presentation of information has evolved and give some idea of the results These four plays will be referred to by the following code

Play	*Location*	*Date*	*Participation*	*Persons per side*
I	Endicott	October 16–17, 1956	Middle management	1
IIA,B	Los Angeles	December 3–4, 1956	Company vice-presidents	2 and 3
IIIA,B	New York	March 6–7, 1957	Company vice-presidents	3
IVA,B	New York	May 2, 1957	Company presidents	4 and 5

Figures 2, 3, and 4 (applicable to Plays I, III, and IV, respectively) indicate the advances made in form design While the basic structure of the game remained essentially the same, the inclusion of income taxes in Play III reduced the growth in assets It is also to be noted that the nature of information, and the manner in which it was obtained, changed in the direction of giving added realism, e g , a change in share-of-the-market information was incorporated Also, the information was presented more nearly in the form to which top management is accustomed

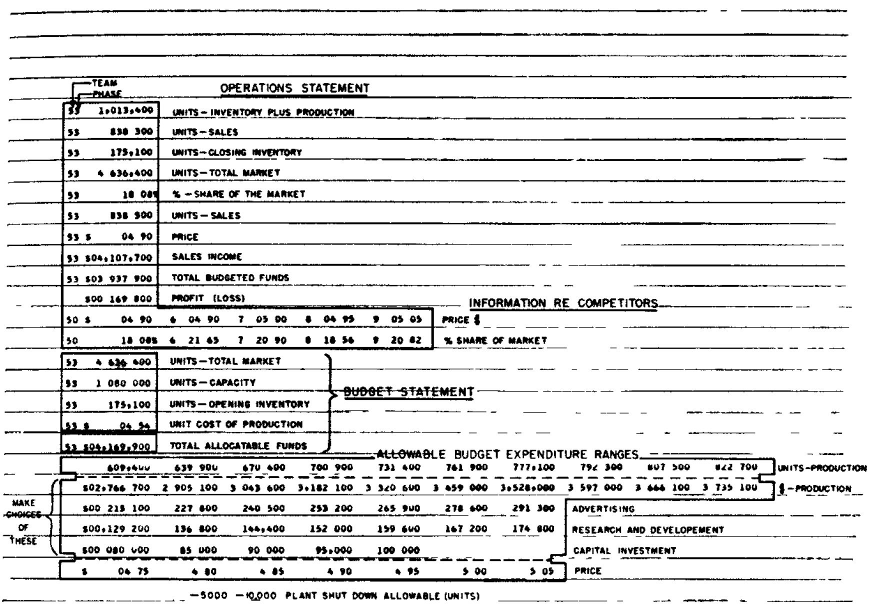
Control Room

The results of each company's actions were charted in the control room These afforded information to the observers and provided the basis for a critique at the end of the game Figure 5 is a typical chart taken from Play IIB, company 1

Total assets (cash and inventory and plant investment), also plotted in the control room, gradually seemed to become the best over-all measure of performance Figures 6–11, inclusive, represent the plot of total assets for various games as indicated.

Critique

At the end of each game, each company was handed the control charts, similar to those presented in Figs 12–17, and a team member was asked to give an explanation of the strategy used during the game and to state what difficulties the group had experienced in its decision making



TEAM
PHASE

OPERATIONS STATEMENT

53	1,013,400	UNITS – INVENTORY PLUS PRODUCTION
53	838 300	UNITS – SALES
53	175,100	UNITS – CLOSING INVENTORY
53	4 636,400	UNITS – TOTAL MARKET
53	18 08%	% – SHARE OF THE MARKET
53	838 300	UNITS – SALES
53 $	04 90	PRICE
53	$04,107,700	SALES INCOME
53	$03 937 900	TOTAL BUDGETED FUNDS
	$00 169 800	PROFIT (LOSS)

INFORMATION RE COMPETITORS

50 $	04 90	6 04 90	7 05 00	8 04 95	9 05 05	PRICE $
50	18 08%	6 21 65	7 20 90	8 18 56	9 20 82	% SHARE OF MARKET

BUDGET STATEMENT

53	4 636 400	UNITS – TOTAL MARKET
53	1 080 000	UNITS – CAPACITY
53	175,100	UNITS – OPENING INVENTORY
53 $	04 56	UNIT COST OF PRODUCTION
53	$04,169,900	TOTAL ALLOCATABLE FUNDS

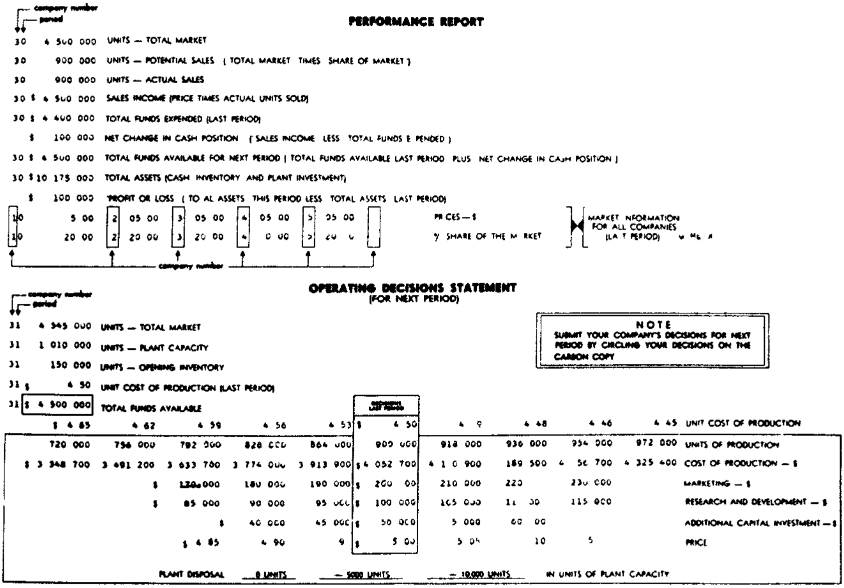
ALLOWABLE BUDGET EXPENDITURE RANGES

609,400	639 900	670 400	700 900	731 400	761 900	777,100	792 300	807 500	822 700	UNITS-PRODUCTION
$02,766 700	2 905 100	3 043 600	3,182 100	3 320 600	3 459 000	3,528,000	3 597 000	3 666 100	3 735 100	$-PRODUCTION
$00 218 100	227 800	240 500	253 200	265 900	278 600	291 300	ADVERTISING			
$00,129 200	136 800	144,400	152 000	159 600	167 200	174 800	RESEARCH AND DEVELOPEMENT			
$00 080 000	85 000	90 000	95,000	100 000			CAPITAL INVESTMENT			
$ 04 75	4 80	4 85	4 90	4 95	5 00	5 05	PRICE			

MAKE ~~CHOICES~~ OF THESE

–5000 –10,000 PLANT SHUT DOWN ALLOWABLE (UNITS)

Fig 2 Game report form, Endicott, Oct 16–17, 1956



company number
period

PERFORMANCE REPORT

30	4 500 000	UNITS — TOTAL MARKET
30	900 000	UNITS — POTENTIAL SALES (TOTAL MARKET TIMES SHARE OF MARKET)
30	900 000	UNITS — ACTUAL SALES
30	$ 4 500 000	SALES INCOME (PRICE TIMES ACTUAL UNITS SOLD)
30	$ 4 400 000	TOTAL FUNDS EXPENDED (LAST PERIOD)
	$ 100 000	NET CHANGE IN CASH POSITION (SALES INCOME LESS TOTAL FUNDS E PENDED)
30	$ 4 500 000	TOTAL FUNDS AVAILABLE FOR NEXT PERIOD (TOTAL FUNDS AVAILABLE LAST PERIOD PLUS NET CHANGE IN CASH POSITION)
30	$10 175 000	TOTAL ASSETS (CASH INVENTORY AND PLANT INVESTMENT)
	$ 100 000	PROFIT OR LOSS (TO AL ASSETS THIS PERIOD LESS TOTAL ASSETS LAST PERIOD)

10 5 00	2 05 00	3 05 00	4 05 00	5 05 00		PR CES — $
10 20 00	2 20 00	3 20 00	4 0 00	5 20 0		% SHARE OF THE M RKET

company number

MARKET NFORMATION FOR ALL COMPANIES (LA T PERIOD)

OPERATING DECISIONS STATEMENT
(FOR NEXT PERIOD)

company number
period

31	4 545 000	UNITS — TOTAL MARKET
31	1 010 000	UNITS — PLANT CAPACITY
31	150 000	UNITS — OPENING INVENTORY
31	$ 4 50	UNIT COST OF PRODUCTION (LAST PERIOD)
31	$ 4 500 000	TOTAL FUNDS AVAILABLE

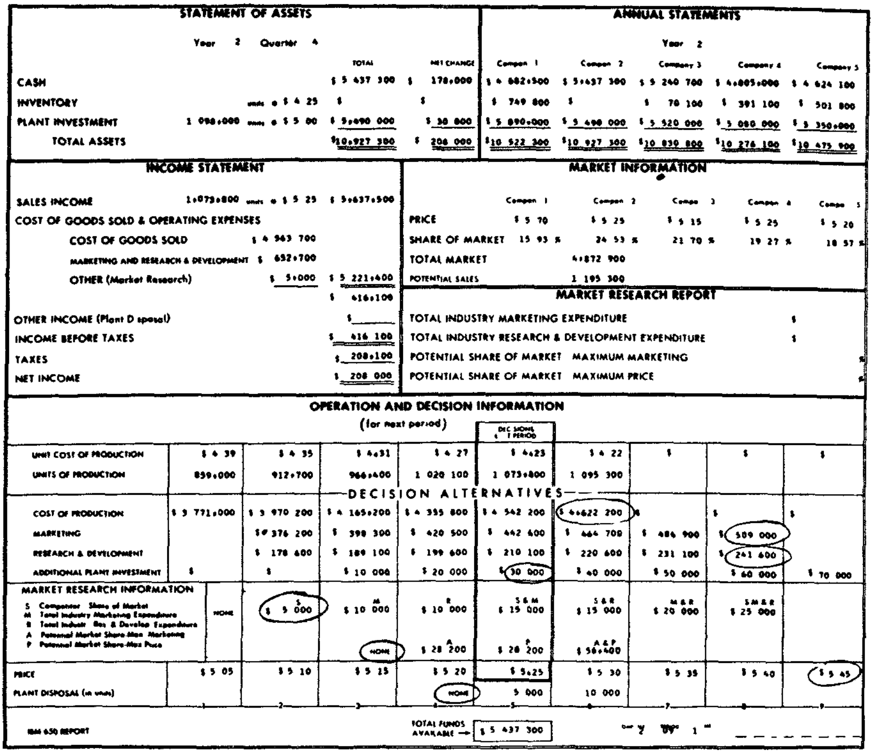
NOTE
SUBMIT YOUR COMPANY'S DECISIONS FOR NEXT PERIOD BY CIRCLING YOUR DECISIONS ON THE CARBON COPY

DEC ON ALTERNATIVES

$ 4 65	4 62	4 59	4 56	4 53	$ 4 50	4 9	4 48	4 46	4 45	UNIT COST OF PRODUCTION
720 000	756 000	792 000	828 000	864 000	900 000	918 000	936 000	954 000	972 000	UNITS OF PRODUCTION
$ 3 348 700	3 491 200	3 633 700	3 774 000	3 913 900	$ 4 052 700	4 1 0 900	189 500	4 56 700	4 325 400	COST OF PRODUCTION — $
		$ 170,000	180 000	190 000	$ 200 00	210 000	220	230 000		MARKETING — $
		$ 85 000	90 000	95 000	$ 100 000	105 000	11 00	115 000		RESEARCH AND DEVELOPMENT — $
			$ 40 000	45 000	$ 50 000	5 000	60 00			ADDITIONAL CAPITAL INVESTMENT — $
		$ 4 85	4 90	9	$ 5 00	5 05	10	5		PRICE

PLANT DISPOSAL 0 UNITS — 5000 UNITS — 10,000 UNITS IN UNITS OF PLANT CAPACITY

Fig 3 Game report form, New York, March 6–7, 1957



STATEMENT OF ASSETS

Year 2 Quarter 4

			TOTAL	NET CHANGE
CASH			$ 5 437 300	$ 178,000
INVENTORY	units @ $ 4 25		$	$
PLANT INVESTMENT	1 098,000 units @ $ 5 00		$ 5,490 000	$ 30 000
TOTAL ASSETS			$10,927 300	$ 206 000

ANNUAL STATEMENTS

Year 2

	Compan 1	Compan 2	Company 3	Company 4	Company 5
CASH	$ 4 882,500	$ 5,437 300	$ 5 240 700	$ 4,805,000	$ 4 624 100
INVENTORY	$ 749 800	$	$ 70 100	$ 391 100	$ 501 800
PLANT INVESTMENT	$ 5 890,000	$ 5 490 000	$ 5 520 000	$ 5 080 000	$ 5 350,000
TOTAL ASSETS	$10 522 300	$10 927 300	$10 830 800	$10 276 100	$10 475 900

INCOME STATEMENT

SALES INCOME	1,073,800 units @ $ 5 25		$ 5,637,500
COST OF GOODS SOLD & OPERATING EXPENSES			
COST OF GOODS SOLD		$ 4 563 700	
MARKETING AND RESEARCH & DEVELOPMENT		$ 652,700	
OTHER (Market Research)		$ 5,000	$ 5 221,400
			$ 416,100
OTHER INCOME (Plant Disposal)			$
INCOME BEFORE TAXES			$ 416 100
TAXES			$ 208,100
NET INCOME			$ 208 000

MARKET INFORMATION

	Compan 1	Compan 2	Compa 3	Compan 4	Compa 5
PRICE	$ 5 70	$ 5 25	$ 5 15	$ 5 25	$ 5 20
SHARE OF MARKET	15 93 %	24 53 %	21 70 %	19 27 %	18 57 %
TOTAL MARKET		4,872 900			
POTENTIAL SALES		1 195 300			

MARKET RESEARCH REPORT

TOTAL INDUSTRY MARKETING EXPENDITURE	$
TOTAL INDUSTRY RESEARCH & DEVELOPMENT EXPENDITURE	$
POTENTIAL SHARE OF MARKET MAXIMUM MARKETING	%
POTENTIAL SHARE OF MARKET MAXIMUM PRICE	%

OPERATION AND DECISION INFORMATION

(for next period)

DECISIONS 1ST PERIOD

UNIT COST OF PRODUCTION	$ 4 39	$ 4 35	$ 4.31	$ 4 27	$ 4.23	$ 4 22	$	$	$
UNITS OF PRODUCTION	859,000	912,700	966,400	1 020 100	1 073,800	1 095 300			
DECISION ALTERNATIVES									
COST OF PRODUCTION	$ 3 771,000	$ 3 970 200	$ 4 165,200	$ 4 355 800	$ 4 542 200	$ 4,622 200	$	$	$
MARKETING		$ 376 200	$ 398 300	$ 420 500	$ 442 600	$ 464 700	$ 486 900	$ 509 000	
RESEARCH & DEVELOPMENT		$ 178 600	$ 189 100	$ 199 600	$ 210 100	$ 220 600	$ 231 100	$ 241 600	
ADDITIONAL PLANT INVESTMENT	$	$	$ 10 000	$ 20 000	$ 30 000	$ 40 000	$ 50 000	$ 60 000	$ 70 000
MARKET RESEARCH INFORMATION S Competitor Share of Market M Total Industry Marketing Expenditure R Total Industr Res & Develop Expenditure A Potential Market Share-Max Marketing P Potential Market Share-Max Price	NONE	S $ 5 000	M $ 10 000 NONE	R $ 10 000 A $ 28 200	S & M $ 15 000 P $ 28 200	S & R $ 15 000 A & P $ 56,400	M & R $ 20 000	S M & R $ 25 000	
PRICE	$ 5 05	$ 5 10	$ 5 15	$ 5 20	$ 5.25	$ 5 30	$ 5 35	$ 5 40	$ 5 45
PLANT DISPOSAL (in units)				NONE	5 000	10 000			
	1	2	3	4	5	6	7	8	9

IBM 650 REPORT

TOTAL FUNDS AVAILABLE → $ 5 437 300

2 1

Fig 4 Game report form, New York, May 2, 1957

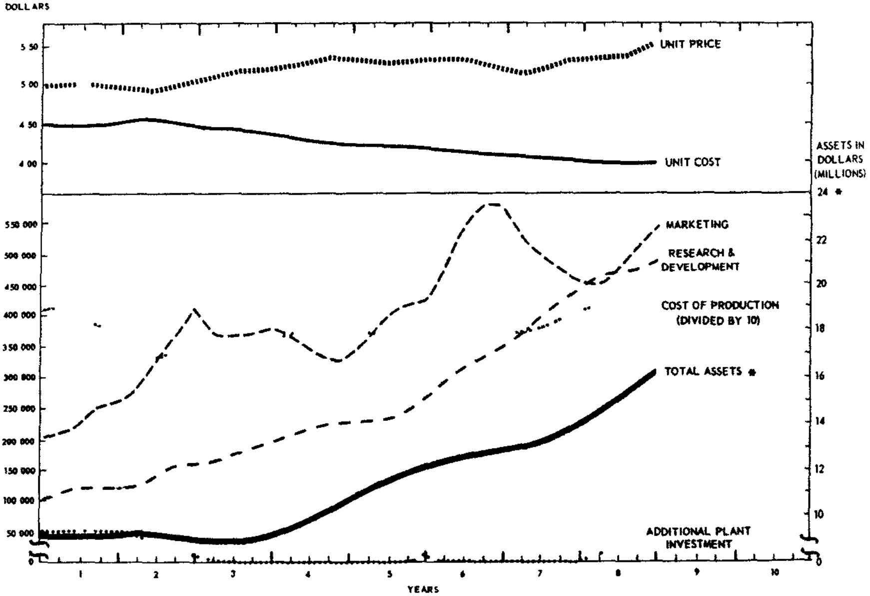


Fig 5 Typical control room chart, Company B1, Game II, Dec 3–4, 1956

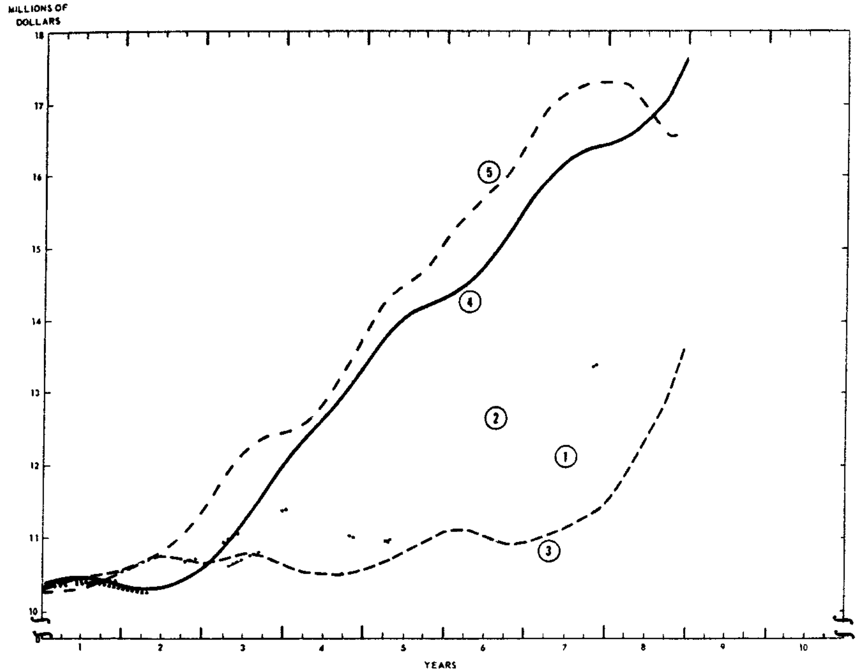


Fig 6. Total assets, Game I, Endicott, Oct 3–4, 1956

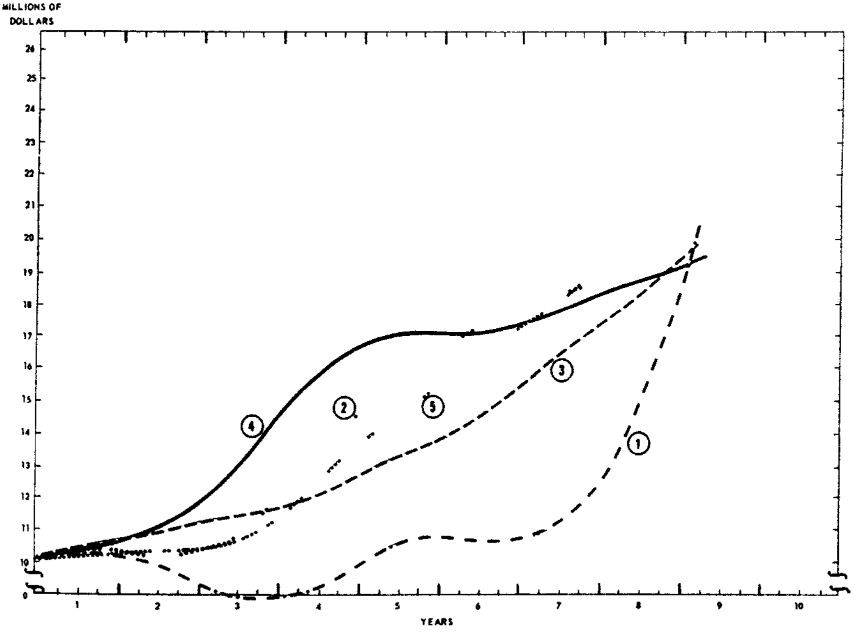


Fig 7 Total assets, Game IIA, Los Angeles, Dec 3–4, 1956

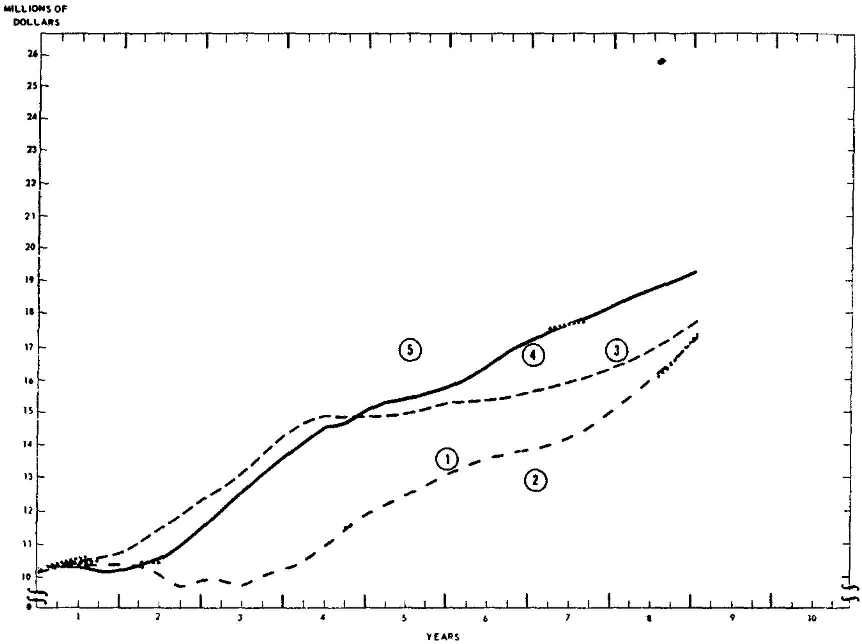


Fig. 8 Total assets, Game IIB, Los Angeles, Dec 3–4, 1956

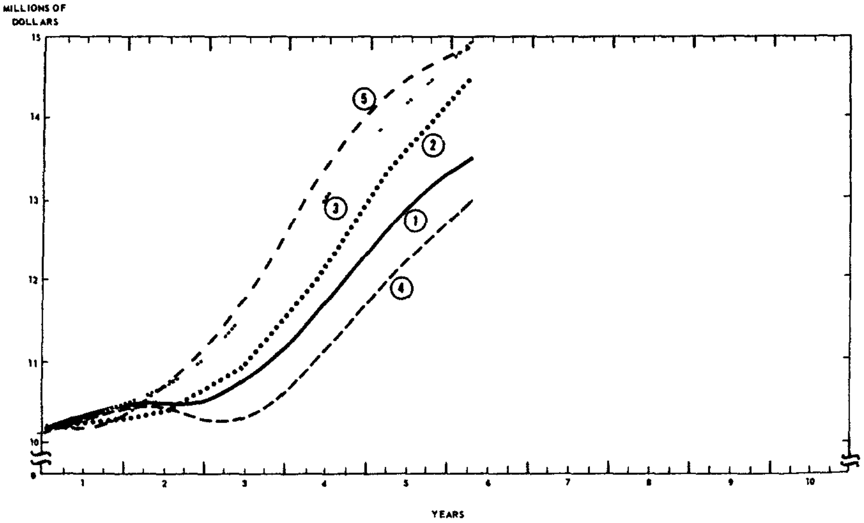


Fig 9 Total assets, Game IIIA, New York, March 6–7, 1957

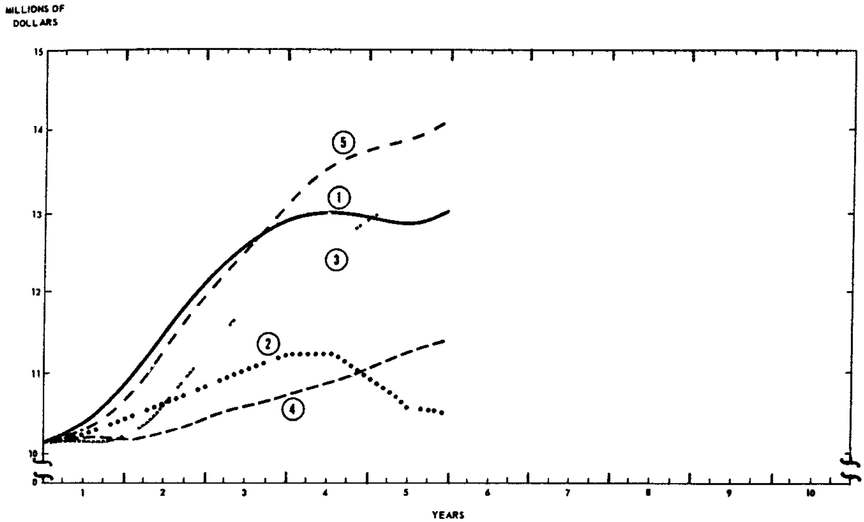


Fig 10 Total assets, Game IIIB, New York, March 6–7, 1957

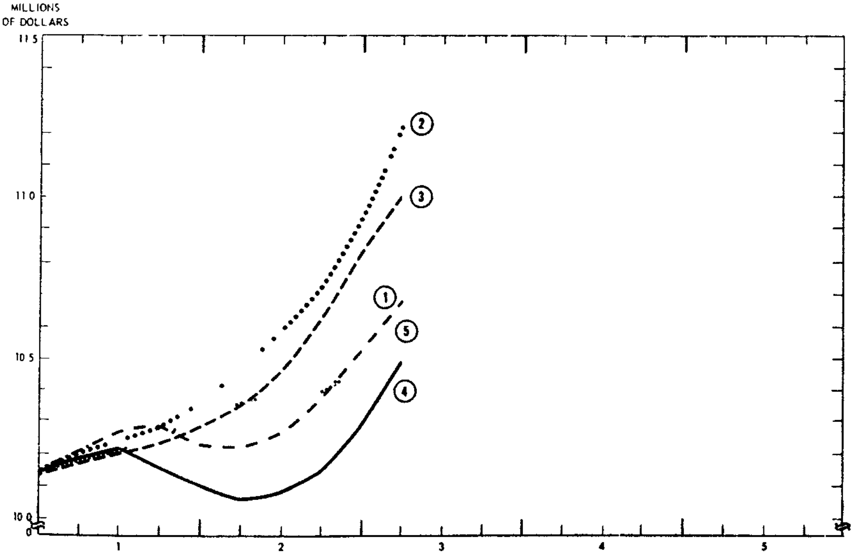


Fig. 11. Total assets, Game IV, New York, May 2, 1957

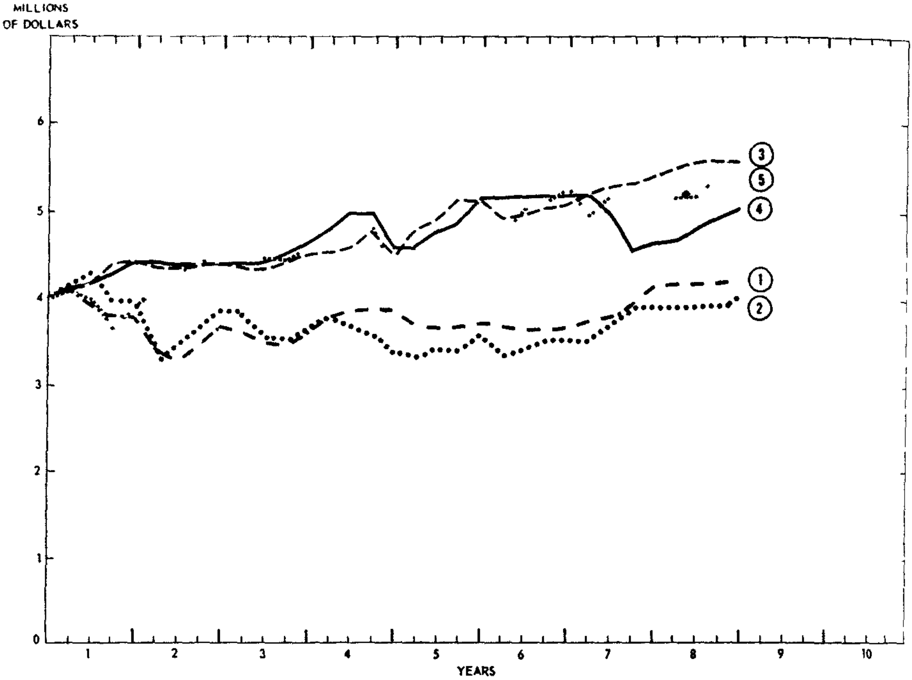


Fig 12 Control chart, costs of production, Game IIB, Los Angeles, Dec 3–4, 1956

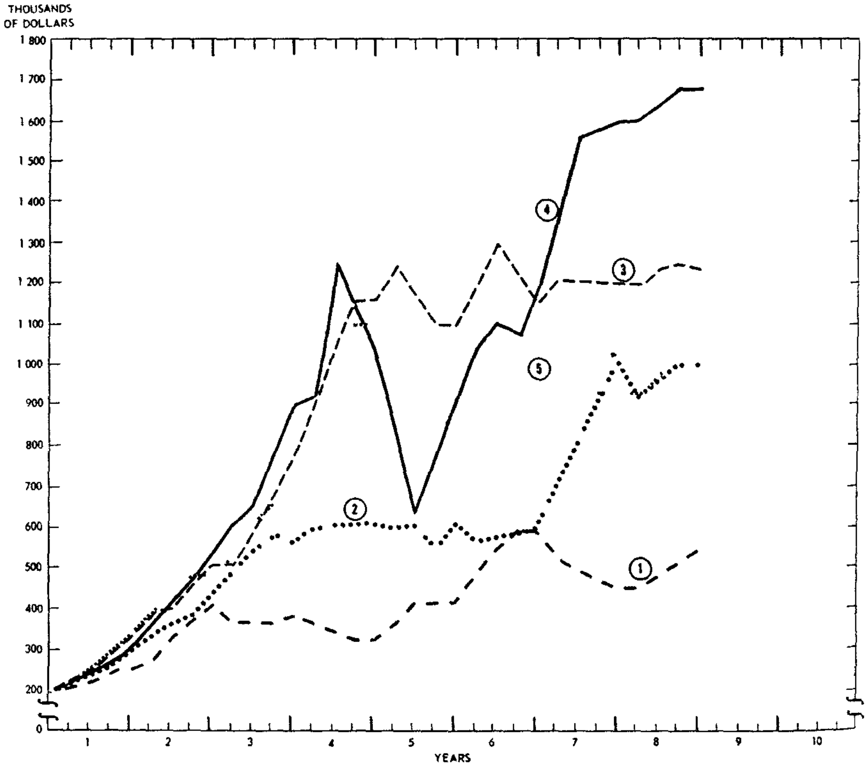


Fig. 13. Control chart, marketing expenditures, Game IIB, Los Angeles, Dec 3–4, 1956

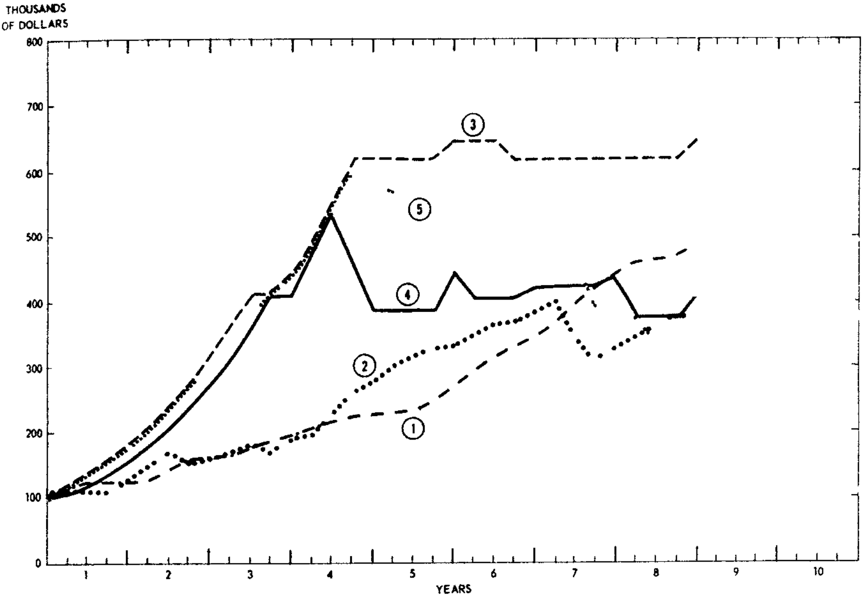


Fig 14. Control chart, research & development expenditures, Game IIB, Los Angeles, Dec 3–4, 1956

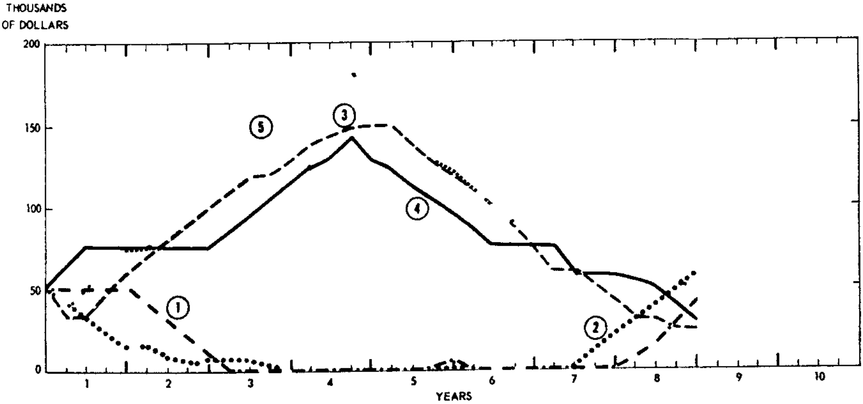


Fig. 15 Control chart, additional plant investment expenditures, Game IIB, Los Angeles, Dec 3–4, 1956

At this point in the research, it is too early to draw any conclusions from the observed performance However, those who have played the game feel that it has great possibilities for improving judgment and reasoning capacity and that it vividly demonstrates the complexity of running a modern business As one participant put it, "The game's great merit lies in reminding the players of the complex interlocking nature of

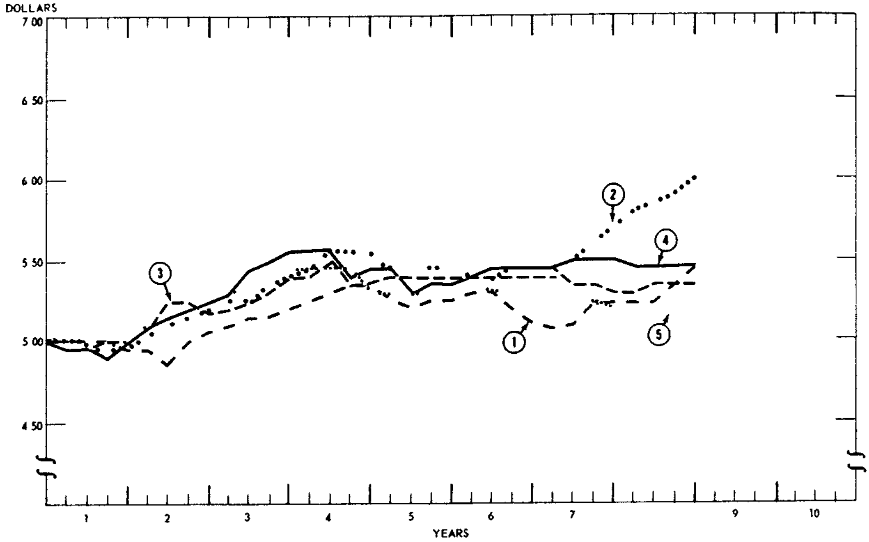


Fig. 16 Control chart, unit prices, Game IIB, Los Angeles, Dec 3–4, 1956

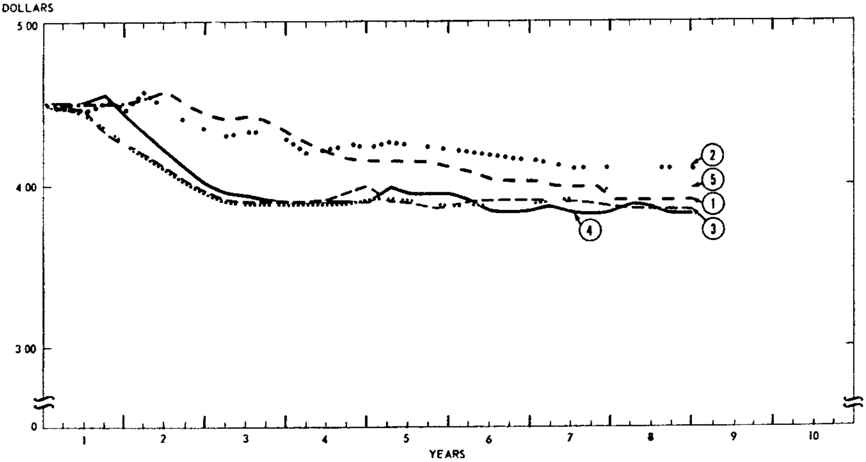


Fig. 17 Control chart, unit costs, Game IIB, Los Angeles Dec 3–4, 1956

the factors that affect most decisions—management by rule of thumb is no longer possible ”

Some participants remarked that the experience suggested ideas to consider in their own businesses, others felt that it would provide a most efficient way of training second-level management in the problems of obtaining a balanced program A comment often heard indicated that the game gave an insight into just what information—that is, what data and reports—one should have to facilitate his own decision-making Other typical comments are "This game really brings out the importance of having facts in decision making It also forceably demonstrates the need for keeping a company's operations in balance ” "The game is not very different from real life It may not be like any particular consumer goods or industrial goods market, but there is a great similarity between the basic things you do in the game and the results that you get They may not happen at the time you think they will or they may happen a lot quicker than you think they should, but they do seem to happen ”

TABLE I

ANALYSIS INFORMATION USAGE—GAME III

Total assets at end of game	Per cent of plays in which information was purchased	
	Combination of $S, M, R^{(a)}$	Combination of $A, P^{(a)}$
	%	%
Game III A		
14,931,000	33	0
14,898,000	46	2
14,497,000	37	9
13,475,000	46	7
12,955,000	19	7
Game III B		
14,137,000	33	2½
13,675,000	60	12½
13,012,000	13	7½
11,384,000	50	10
10,506,000	55	12½

(a) See Fig 1 for greater detail S = competition share of the market, M = total industry marketing expenditure, R = total industry research and development expenditure, A = potential market share—maximum marketing, P = potential market share—maximum price

As has been seen, the game provides the researcher with a laboratory tool to use in observing the decision-making process For example, it is interesting to note the use which players in Plays IIIA and IIIB made of the opportunity to purchase market information Table I indicates the percentage of times that a given company purchased "Market Research Information" of various types—with the various companies arrayed in order of decreasing total assets at the end of the game

Many other types of statistical analysis can be carried out through the use of gaming techniques under controlled conditions involving many plays It is hoped that the abbreviated results here presented will be suggestive of the potential research uses of simulation techniques

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SUGGESTIONS FOR GENERAL READING

An enormous amount of work has been done in the field of simulation and gaming and related disciplines For those entering this field and wishing to acquire an over-all perspective, we suggest the following books and articles

Simulation and Operational Gaming

C J Thomas and W L Deemer, Jr , "The Role of Operational Gaming in Operations Research," *Opns Res* **5,** 1–27 (1957)

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Learning Theory

R Bush and F Mosteller, *Stochastic Models for Learning*, Wiley, New York, 1955
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